Topic 6: The Arithmetic of Financial Engineering

Basic Equation

- A + B = C

(This plus that equals another thing)

- Let's practice with bonds

The DCF approach in general form:

- Given an efficient market, NPV is zero for a securities transaction
- Therefore, today's price equals PV of all expected future cash flows

$$NPV = -P_0 + \sum_{i=1}^{n} \frac{\overline{C}_i}{(1+R)^i} = 0$$

$$\therefore P_0 = \sum_{i=1}^{n} \frac{\overline{C}_i}{(1+R)^i}$$

The DCF approach to coupon bonds:

- **Computing price, with a known required rate of return:**
- **Computing yield-to -maturity**
  - **equals the rate implied by the market price**

$$P_0 = \frac{\text{Face Value}}{(1+\text{R})^n} + \sum_{i=1}^{n} \frac{\text{Coupon Pmt}}{(1+R)^i}$$

$$\text{Market Price} = \frac{\text{Face Value}}{(1+\text{R})^n} + \sum_{i=1}^{n} \frac{\text{Coupon Pmt}}{(1+R)^i}$$

Example 1: Computing Price

- Face Value is $1,000
- Coupon rate is 7%
- Market rate is 8% (semi-annual)
- Maturity is 20 years

- Then FV is 1000
- PMT is 35
- Interest is 8
- P/YR is 2
- N is 40
- Compute PV
- = $901.04
- Negative sign in display reflects sign convention

Example 2: Computing Yield

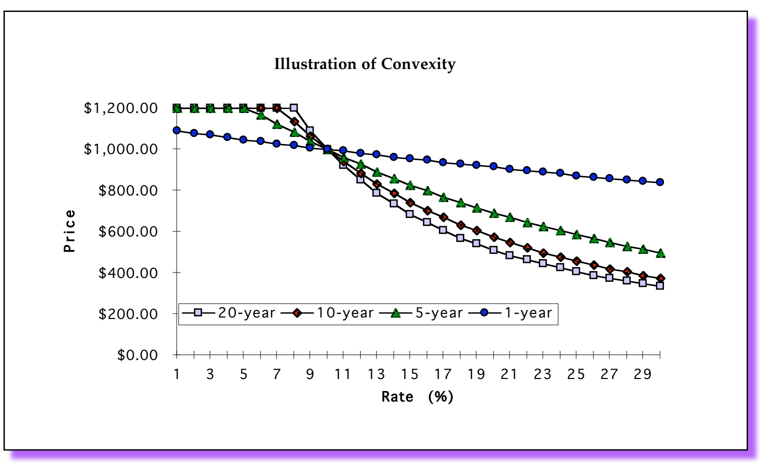
- Face Value is $1,000
- Coupon rate is 7%
- Maturity is 20 years (semi-annual)
- Price is $815.98

- Then FV is 1000
- PMT is 35
- N is 40
- P/YR is 2
- PV is -815.98
- Compute interest
- = 9.00%

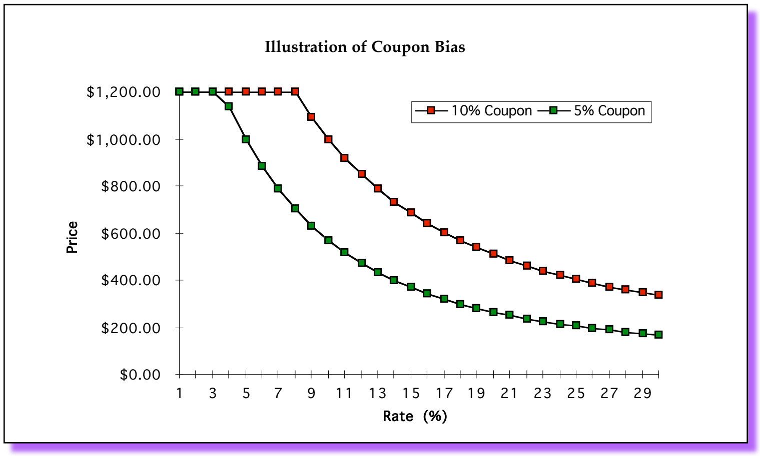
Table Illustrating Coupon Bias and Convexity

	old rate	new rate	old price	new price	capital gain (loss)	relative change
20-year, 10% bonds	12%	15%	$849.54	$685.14	($164.40)	-19.35%
	12%	9%	$849.54	$1,092.01	$242.47	+28.54%
	8%	10%	$1,197.93	$1,000.00	($197.93)	-16.52%
	8%	6%	$1,197.93	$1,462.30	$264.37	+22.07%
	17%	18%	$603.99	$569.71	($34.28)	-5.68%
	17%	16%	$603.99	$642.26	$38.27	+6.34%
10-year, 10% bonds	12%	15%	$885.30	$745.14	($140.16)	-15.83%
	12%	9%	$885.30	$1,065.04	$179.74	+20.30%
	8%	10%	$1,135.90	$1,000.00	($135.90)	-11.96%
	8%	6%	$1,135.90	$1,297.55	$161.65	+14.23%
	17%	18%	$668.78	$634.86	($33.92)	-5.07%
	17%	16%	$668.78	$705.46	$36.68	+5.48%
20-year, 5% bonds	12%	15%	$473.38	$370.28	($103.10)	-21.78%
	12%	9%	$473.38	$631.97	$158.59	+33.50%
	8%	10%	$703.11	$571.02	($132.09)	-18.79%
	8%	6%	$703.11	$884.43	$181.32	+25.79%
	17%	18%	$321.13	$300.77	($20.36)	-6.34%
	17%	16%	$321.13	$344.15	$23.02	+7.17%
10-year, 5% bonds	12%	15%	$598.55	$490.28	($108.27)	-18.09%
	12%	9%	$598.55	$739.84	$141.29	+23.61%
	8%	10%	$796.15	$688.44	($107.71)	-13.53%
	8%	6%	$796.15	$925.61	$129.46	+16.26%
	17%	18%	$432.20	$406.64	($25.56)	-5.91%
	17%	16%	$432.20	$460.00	$27.80	+6.43%

Convexity



Coupon Bias



Risk factors for bondholders:

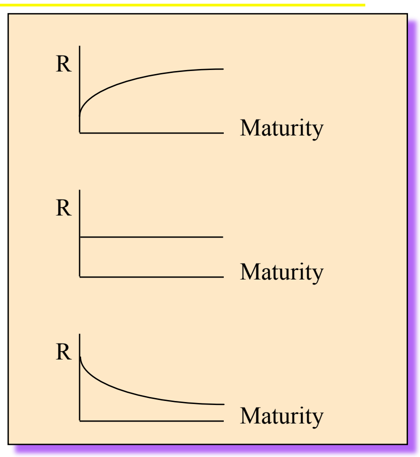
- Purchasing power risk
- Interest rate risk
- Reinvestment risk
- Default risk

The yield curve:

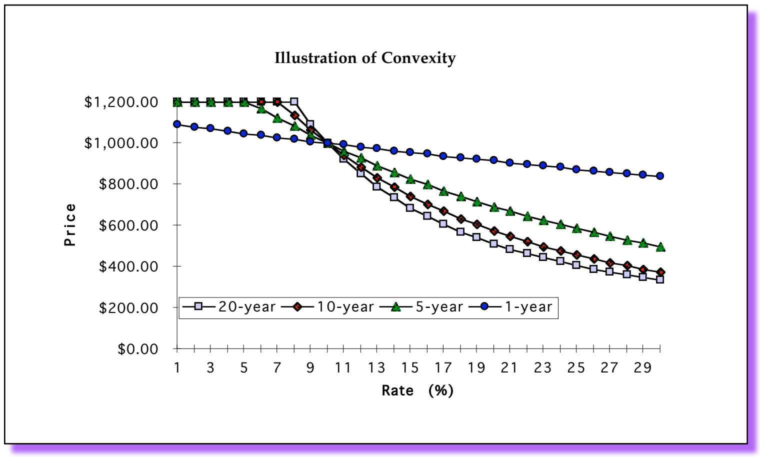
- **R = r + inflation adjustment + risk adjustment.**
- **Inflation adjustment:**
  R = r + i + ri
  r = (R–i)/(1+i)
- **Three theories to explain the yield curve**
  – **Liquidity Premium Theory**
  – **Pure Expectations Theory (PET)**
    - **also known as the Rational Expectations Theory**
    - **easily remembered as the "Pet Rat"**
  – **Preferred Habitat Theory**

Let's see how different theories explain what we observe:

- **Upward sloping yield curve**
- **Flat yield curve**
- **Downward sloping yield curve**



Convexity



McCauley's Duration:

Weighted average maturity

0	1	2	3
$300	**$110**	**$121**	**$133.10**
Cash flows Discounted at 10%	**$100**	**$100**	**$100**

$$\text{Duration} = 1\left(\frac{100}{300}\right) + 2\left(\frac{100}{300}\right) + 3\left(\frac{100}{300}\right)$$

$$\text{Duration} = 2$$

Coupon Stripping

- **Stripped zeroes of different maturities can be used to construct the yield curve**
- **Better than using duration**

Another Approach to Yield Curve

<u>Bond A</u>
- No coupon
- 6 months to maturity
- Price: $98.52
- Yield: 3%

<u>Bond B</u>
- 3% coupon (2 pmts/yr)
- 1 year to maturity
- Price: $99.52

Let's use this information to find pure one-year rate implied in the second bond

- 1st payment is $1.50
- 6-month rate is 3%
- PV is $1.48

- Final payment: $101.50
- Cost is $99.52 – $1.48
- = $98.04

Implied one-year rate is 3.5% APR (2 P/YR)

Finding the Yield Curve

	0	1	2
Start:	$\$99.^{52}$	$\$1.^{50}$	$\$101.^{50}$
Simplified:	$\$1.^{48}$ ←	$\$1.^{50}$	
Net:	$\$98.^{04}$	0	$101.^{50}$

Implied one-year rate is 3.5% APR (2 P/YR)

Another Approach to Yield Curve

Background data:
- 6-month rate: 3%
- 1-year rate: 3.5%

Bond C:
- 3% coupon
- 18 months to maturity
- Price: $\$99.^{10}$

Let's use this information to find pure 18-month rate implied in the second bond

- 1st pmt has PV $\$1.^{48}$
- 2nd pmt has PV $\$1.^{45}$
- Final payment: $\$101.^{50}$
- Cost is $\$99.^{10} - \$1.^{48} - \$1.^{45} = \$96.^{17}$

Implied 18-month rate is $3.^{63}$% APR (2 P/YR)

Finding the Yield Curve

	0	1	2	3
Start:	$99.10	$1.50	$1.50	$101.50
Simplified:	$1.48 ←	$1.50		
	$1.45 ←		$1.50	
Net:	$96.17	0	0	101.50

Implied 18-month rate is 3.63% APR (2 P/YR)

Another Approach to Yield Curve

Background data:

- 6-month rate: 3%
- 1-year rate: 3.5%
- 18-month rate: 3.63%

Bond D:

- 3% coupon
- 2 years to maturity
- Price: $98.67

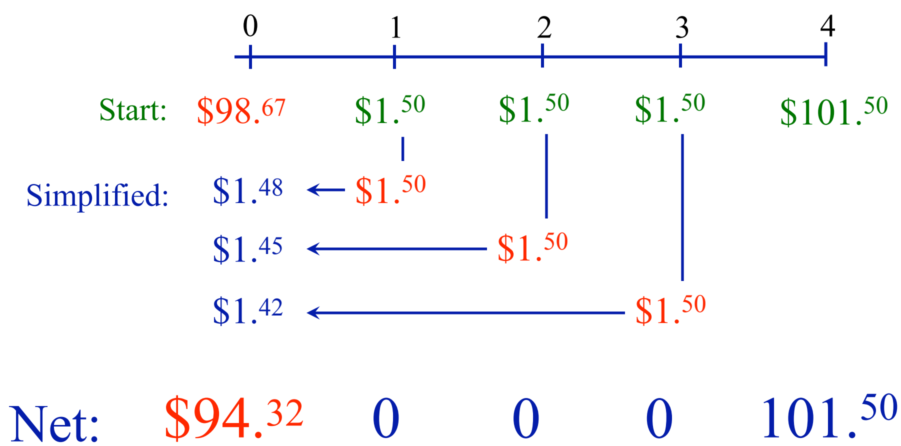
Let's use this information to find pure two-year rate implied in the second bond

- 1st pmt has PV $1.48
- 2nd pmt has PV $1.45
- 3rd pmt has PV $1.42

- Final payment: $101.50
- Cost is $98.67 – $1.48 – $1.45 – $1.42 = $94.32

Implied 2-year rate is 3.70% APR (2 P/YR)

Finding the Yield Curve



Implied 2-year rate is 3.70% APR (2 P/YR)

Deriving Implied Forward Rates

Bond A

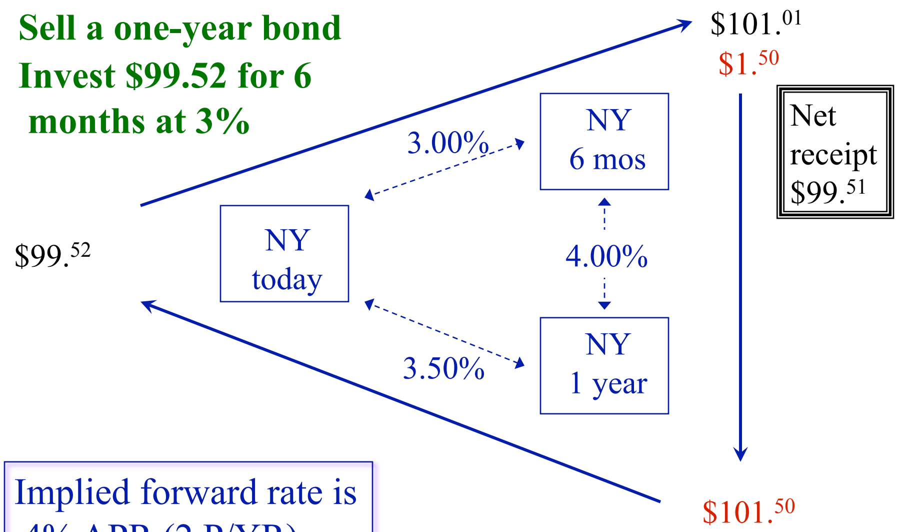
- No coupon
- 6 months to maturity
- Price: $98.52
- Yield: 3%

Bond B

- 3% coupon
- 1 year to maturity
- Price: $99.52
- Yield: 3.49%

Deriving Implied Forward Rates

- **Sell a one-year bond**
- **Invest $99.52 for 6 months at 3%**



Implied forward rate is 4% APR (2 P/YR)

Lock in a future loan

Bond A:

- 3% coupon
- 1 year to maturity
- Price: $99.52

Bond B:

- 3% coupon
- 18 months to maturity
- Price: $99.10

Buy a one-year bond and sell an 18-month bond

Net payment $99.52 - $99.10 = $0.42

In 6 months:

- Receive: $1.50
- Pay: $1.50
- Net zero

In 1 year:

- Receive: $101.50
- Pay: $1.50
- Net $100

In 18 months:

- Pay: $101.50

Paid $0.42 to contract a $100 loan in 1 year at 3% APR

Lock in a future loan

	0	1	2	3
Buy	$\$99.^{52}$	$\$1.^{50}$	$\$101.^{50}$	
Sell	$\$99.^{10}$	$\$1.^{50}$	$\$1.^{50}$	$\$101.^{50}$
Net	$\$0.^{42}$	0	\$100	$\$101.^{50}$

Paid $\$0.^{42}$ to contract a \$100 loan in 1 year at 3% APR

Lock in a future loan

- This is like a futures contract with futures price 3% (underlying is a loan to be delivered six months from today
- This is the same as a long call and a short put with exercise price 3% and expiration in 6 months
- So we know that Call minus Put = \$0.42
- From this observation we could derive insight into uncertainty about interest rates and slope of the yield curve

Arbitrage in Bond Market

- 6% coupon
- 10 year
- Price: $80.00
- Yield: 9.09%

- 8% coupon
- 10 year
- Price: $90.00
- Yield: 9.58%

- 10% coupon
- 10 year
- Price: $104.00
- Yield: 9.37%

Let's Arbitrage!

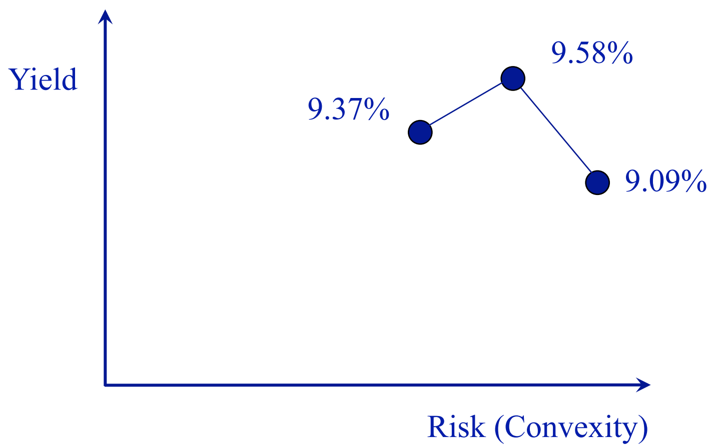
- Sell a 6%: receive $80.00
- Sell a 10%: receive $104.00
- Buy two 8s: pay $180.00
- Net: receive $4.00

Arbitrage

	0	1	2	…	19	20
Buy	$180.00	$8	$8	……..	$8	$208
Sell	$80.00	$3	$3	……..	$3	$103
Sell	$104.00	$5	$5	……..	$5	$105
Net	$4.00	0	0	……….	0	0

NPV is always positive

What's wrong?



Arbitrage in Bond Market

- 4% coupon
- 8 year
- Price: $90.40
- Yield: 5.50%

- 4% coupon
- 7 year
- Price: $88.70
- Yield: 6.00%

- 4% coupon
- 6 year
- Price: $94.87
- Yield: 5.00%

Let's Arbitrage!

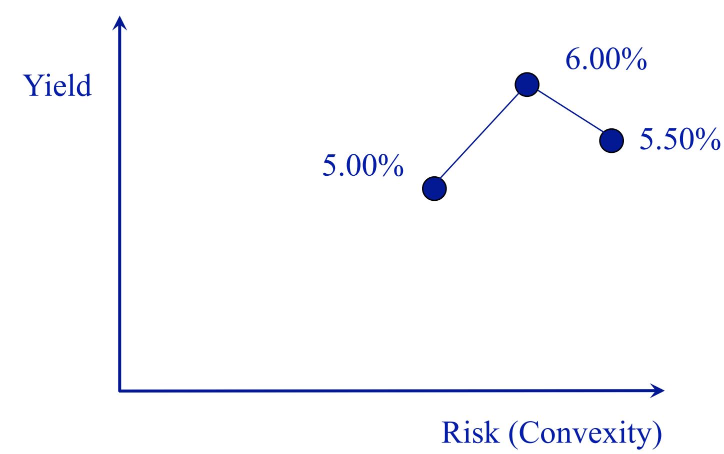
- Sell a 6-year: receive $94.87
- Sell an 8-year: receive $90.40
- Buy two 7-years: pay $177.40
- Net: receive $7.87

Arbitrage

	0	1		11	12	13	14	15	16
Buy	$177.40	$4	····	$4	$4	$4	$204		
Sell	$94.87	$2	····	$2	$102				
Sell	$90.40	$2	····	$2	$2	$2	$2	$2	$102
Net	$7.87	0	····	0	$100	$2	$202	$2	$102

NPV is always positive

What's wrong?



Arbitrage in Bond Market

- 4% coupon
- 8 year
- Price: $90.40
- Yield: 5.00%

- 4% coupon
- 7 year
- Price: $94.86
- Yield: 4.88%

- 4% coupon
- 6 year
- Price: $94.87
- Yield: 5.00%

Let's Arbitrage!

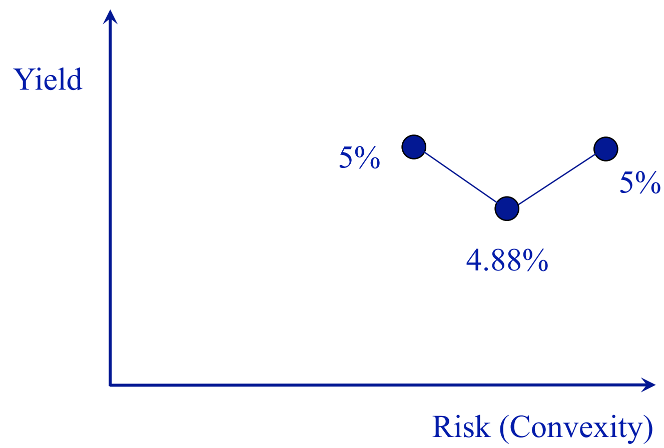
- Buy an 8-year: pay $90.40
- Buy a 6-year: pay $94.87
- Sell two 7s: receive $189.72
- Net: receive $4.45

Arbitrage

	0	1		11	12	13	14	15	16
Sell	$189.72	$4	····	$4	$4	$4	$204		
Buy	$94.87	$2	····	$2	$102				
Buy	$90.40	$2	····	$2	$2	$2	$2	$2	$102
Net	$4.45	0	····	0	$100	$2	$202	$2	$102

NPV is always positive

What's wrong?

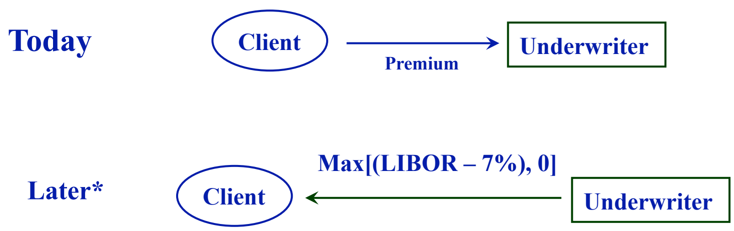


IRRM Products

- Listed Products
  - Futures
  - Options
- Custom Products
  - Swaps
  - Caps
  - Floors
  - Collars

Caps

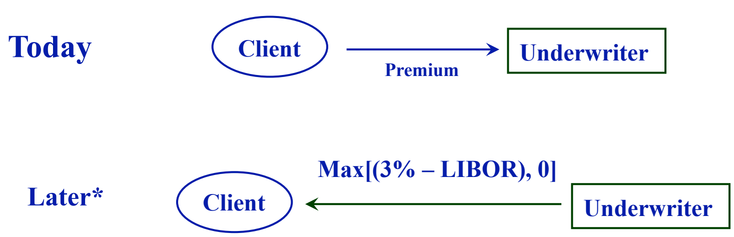
Illustration of a 7% Interest Rate Cap on LIBOR



*Payments are made periodically (say, monthly or quarterly) over the life of the contract, with rates appropriately adjusted for the number of periods per year

Floors

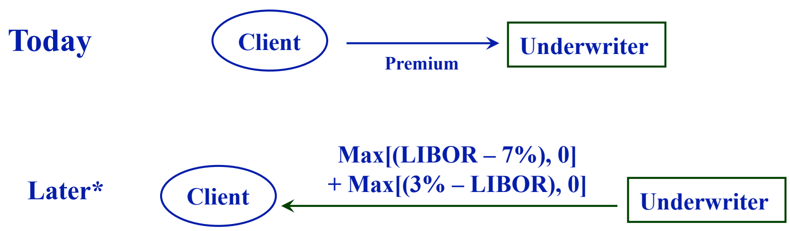
Illustration of a 3% Interest Rate Floor on LIBOR



*Payments are made periodically (say, monthly or quarterly) over the life of the contract, with rates appropriately adjusted for the number of periods per year

Collars

Illustration of a 3,7 Collar on LIBOR



*Payments are made periodically (say, monthly or quarterly) over the life of the contract, with rates appropriately adjusted for the number of periods per year